Statistical Tolerance Limits

Statistical Tolerance limits provide you with a range in which some percentage of all your products will lie. Here is an example:

Suppose you are making ball bearings. You know that you have response variation in your process, so you know that the ball bearings produced over time will vary in diameter. You calculate the average and standard deviation from the diameter measurements of ten randomly selected ball bearings from your process. You find an average diameter of 0.125 inches with a standard deviation of 0.004 inches. You would like to know lower and upper limits on the diameters of ball bearings produced by your process.

As is often the case in the real world, you cannot define these limits for ALL of the ball bearings made. You can, though, determine limits on 99% of all ball bearings made. These limits are called "Tolerance Limits on 99% of the Population." As is also often the case in the real world, you cannot be 100% confident that these limits are correct. However, you can be 95% confident that they are correct. These limits are then called "95% Tolerance limits on 99% of the Population."

If your diameter measurements fall in bell-shaped piles (are "Normally Distributed") then you can calculate 95% Tolerance Limits on 99% of the Population using the simple formula,

Lower Tolerance Limit = Average - Ks

Upper Tolerance Limit = Average + Ks

s is the standard deviation and K is a number from the table below.

For the ball bearing example,

Lower Tolerance Limit = 0.125 - 4.433 * 0.004 = 0.107

Upper Tolerance Limit = 0.125 + 4.433 * 0.004 = 0.143

So, over the course of ball bearing production we would expect 99% of all of the ball bearings produced to have diameters between 0.107 inches and 0.143 inches. We are also 95% confident that these limits are correct.

How to read the table: First decide on the confidence level you want. The columns are grouped by confidence level. (95% is the usual confidence level.) Second decide what percentage of all of the future products you will make should be included between the limits. Choose the column for this percentage from the three columns under your confidence level. (99% is normal.) Third you will move down the column just chosen until you find the row with the number of replicates you ran.

Table of K Values for Calculating Tolerance Limits

	90% Confidence			95% Confidence			99% Confidence		
	95%	99%	99.9%	95%	99%	99.9%	95%	99%	99.9%
2	18.800	24.167	30.227	37.674	48.430	60.573	188.491	242.300	303.054
3	6.919	8.974	11.309	9.916	12.861	16.208	22.401	29.055	36.616
4	4.943	6.440	8.149	6.370	8.299	10.502	11.150	14.527	18.383
5	4.152	5.423	6.879	5.079	6.634	8.415	7.855	10.260	13.015
6	3.723	4.870	6.188	4.414	5.775	7.337	6.345	8.301	10.548
7	3.452	4.521	5.750	4.007	5.248	6.676	5.488	7.187	9.142
8	3.264	4.278	5.446	3.732	4.891	6.226	4.936	6.468	8.234
9	3.125	4.098	5.220	3.532	4.631	5.899	4.550	5.966	7.600
10	3.018	3.959	5.046	3.379	4.433	5.649	4.265	5.594	7.129
15	2.713	3.562	4.545	2.954	3.878	4.949	3.507	4.605	5.876
20	2.564	3.368	4.300	2.752	3.615	4.614	3.168	4.161	5.312
25	2.474	3.251	4.151	2.631	3.457	4.413	2.972	3.904	4.985
30	2.413	3.170	4.049	2.549	3.350	4.278	2.841	3.733	4.768
35	2.368	3.112	3.974	2.490	3.272	4.179	2.748	3.611	4.611
40	2.334	3.066	3.917	2.445	3.213	4.104	2.677	3.518	4.493
45	2.306	3.030	3.871	2.408	3.165	4.042	2.621	3.444	4.399
50	2.284	3.001	3.833	2.379	3.126	3.993	2.576	3.385	4.323
55	2.265	2.976	3.801	2.354	3.094	3.951	2.538	3.335	4.260
60	2.248	2.955	3.774	2.333	3.066	3.916	2.506	3.293	4.206
65	2.235	2.937	3.751	2.315	3.042	3.886	2.478	3.257	4.160
70	2.222	2.920	3.730	2.299	3.021	3.859	2.454	3.225	4.120
75	2.211	2.906	3.712	2.285	3.002	3.853	2.433	3.197	4.084
80	2.202	2.894	3.696	2.272	2.986	3.814	2.414	3.173	4.053
85	2.193	2.882	3.682	2.261	2.971	3.795	2.397	3.150	4.024
90	2.185	2.872	3.669	2.251	2.958	3.778	2.382	3.130	3.999
95	2.178	2.863	3.657	2.241	2.945	3.763	2.368	3.112	3.976
100	2.172	2.854	3.646	2.233	2.934	3.748	2.355	3.096	3.954
110	2.160	2.839	3.626	2.218	2.915	3.723	2.333	3.066	3.917
120	2.150	2.826	3.610	2.205	2.898	3.702	2.314	3.041	3.885
130	2.141	2.814	3.595	2.194	2.883	3.683	2.298	3.019	3.857
140	2.134	2.804	3.582	2.184	2.870	3.666	2.283	3.000	3.833
150	2.127	2.795	3.571	2.175	2.859	3.652	2.270	2.983	3.811
160	2.121	2.787	3.561	2.167	2.848	3.638	2.259	2.968	3.792
170	2.116	2.780	3.552	2.160	2.839	3.527	2.248	2.955	3.774
180	2.111	2.774	3.543	2.154	2.831	3.616	2.239	2.942	3.759
190	2.106	2.768	3.536	2.148	2.823	3.606	2.230	2.931	3.744
200	2.102	2.762	3.529	2.143	2.816	3.597	2.222	2.921	3.731
250	2.085	2.740	3.501	2.121	2.788	3.561	2.191	2.880	3.678
300	2.073	2.725	3.481	2.106	2.767	3.535	2.169	2.850	3.641
400	2.057	2.703	3.453	2.084	2.739	3.499	2.138	2.809	3.589
500	2.046	2.689	3.434	2.070	2.721	3.475	2.117	2.783	3.555
600	2.038	2.678	3.421	2.060	2.707	3.458	2.102	2.763	3.530
700	2.032	2.670	3.411	2.052	2.697	3.445	2.091	2.748	3.511
800	2.027	2.663	3.402	2.046	2.688	3.434	2.082	2.736	3.495
900	2.023	2.658	3.396	2.040	2.682	3.426	2.075	2.726	3.483
1000	2.019	2.654	3.390	2.036	2.676	3.418	2.068	2.718	3.472
Inf.	1.960	2.576	3.291	1.960	2.576	3.291	1.960	2.576	3.291

Values in the table are from, "Experimental Statistics," by Mary Natrella, currently out of print.

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